The most general uniform linear transmission line has four primary descriptive constants:

1. The loop resistance per unit length, R.
2. The loop inductance per unit length, L.
3. The shunt conductance per unit length, G.
4. The shunt capacitance per unit length, C.

If there is a voltage $\underline{e}$ between conductors and it is changing at a rate $\frac{\partial e}{\partial t}$ there will be a shunt current di_s in a length dx given by $di_s = (Ge + C\frac{\partial e}{\partial t})\, dx$.

This current is robbed from the current entering the element dx. Hence for the current, i, in the line we have

$$\frac{\partial i}{\partial x} = -\frac{\partial i_s}{\partial x} = -(Ge + C\frac{de}{dt}) \qquad (1)$$

Similarly, the current $\underline{i}$ in the line, changing at a rate $\frac{di}{dt}$ causes a series voltage de_s in the length dx given by

$$de_s = (Ri + L\frac{\partial i}{\partial t})\, dx.$$

The voltage at the output of dx is the input voltage to dx less this series voltage. Thus for the voltage, e, between conductors we have

$$\frac{\partial e}{\partial x} = -\frac{\partial e_s}{\partial x} = -(Ri + L\frac{\partial i}{\partial t}) \qquad (2)$$

By differentiating (1) and (2) and substituting we find:

$$\frac{\partial^2 e}{\partial x^2} = LC\frac{\partial^2 e}{\partial t^2} + (RC + LG)\frac{\partial e}{\partial t} + RGe \qquad (3)$$

$$\frac{\partial^2 i}{\partial x^2} = LC\frac{\partial^2 i}{\partial t^2} + (RC + LG)\frac{\partial i}{\partial t} + RGi \qquad (4)$$

Equations (3) and (4) are one dimensional wave equations the direct general solution of which is difficult. However, two cases admit simple solutions:

a) The "distortionless" case, where $\frac{R}{L} = \frac{G}{C}$

b) The "harmonic" case, where the excitation is sinusoidal.

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The harmonic solution then permits complete specific solutions since the spectrum of the wave can be found at any point on the line if the input spectrum is known, and the spectrum defines the time function. However, let us first consider the "distortionless" case (a).

a) (3) and (4) are of the same form, so let's just talk about (3). The general solution of (3) <u>under the condition $\frac{R}{L} = \frac{G}{C}$ is:</u>

$$e = A e^{-\alpha x} f(x - vt) + B e^{\alpha x} f(x + vt) \qquad (5)$$

where $\alpha = \sqrt{RG}$, $v = \dfrac{1}{\sqrt{LC}}$, and A and B are arbitrary. This solution can be verified by substitution.

What does (5) say? The first term represents a wave traveling to the right (x increasing) at a velocity $\underline{v}$ and dying out at a rate determined by α, but <u>without changes in shape.</u> Similarly, the second term represents a wave traveling to the left (x decreasing) at a velocity v, and also dying out at a rate determined by α, and also <u>without change of shape.</u> Hence the name "distortionless" for this case. Since the wave shape doesn't change, the spectrum amplitude merely decreases with distance equally for all frequencies; i.e., the line attenuation is independent of frequency, the phase shift linear.

Associated with each of these voltage waves is a current wave of the same shape, i.e.:

$$i = C e^{-\alpha x} f(x-vt) + D e^{\alpha x} f(x+vt). \qquad (6)$$

Substitution in (1) and (2) will reveal that

$$\frac{A}{C} = -\frac{B}{D} = \sqrt{\frac{L}{C}} \left(= \sqrt{\frac{R}{G}}\right) \qquad (7)$$

Thus, for the distortionless line the ratio of instantaneous voltage to instantaneous current for any wave is a resistance $Z_0 = \sqrt{\dfrac{L}{C}}$.

b) Things aren't too different in the harmonic case.

We let
$$\begin{aligned} e &= E\, e^{j\omega t}) \\ i &= I\, e^{j\omega t}) \end{aligned} \quad \text{so that} \quad \begin{aligned} \frac{de}{dt} &= (j\omega)e \\ \frac{di}{dt} &= (j\omega)i \end{aligned}$$

And get

$$\frac{dE}{dx} = (R + j\omega L)\ I \qquad (8)$$

$$\frac{dI}{dx} = (G + j\omega C)\ E \qquad (9)$$

$$\frac{d^2E}{dx^2} = \left[RG + j\omega (RC + LG) - \omega^2 LC \right] E \tag{10}$$

$$\frac{d^2I}{dx^2} = \left[RG + j\omega (RC + LG) - \omega^2 LC \right] I \tag{11}$$

We then find as the solutions:

$$E = Ae^{-\gamma x} + Be^{\gamma x} \tag{12}$$

$$I = Ce^{-\gamma x} + De^{\gamma x} \tag{13}$$

Where

$$\gamma^2 = \left[RG + j\omega (RC + LG) - \omega^2 LC \right]$$
$$= (R + j\omega L)(G + j\omega C).$$

That is

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \tag{14}$$

$$= \alpha + j\beta$$

γ is called the <u>propagation</u> constant. It tells what happens to the amplitude and phase of a sinusoid as it propagates down the line. It consists of two parts: α, the <u>attenuation constant</u> (nepers/unit length)

β, the phase constant (radians/unit length).

Thus:

$$e^{-\gamma x} = e^{-\alpha x - j\beta x} = e^{-\alpha x} \quad \times \quad e^{-j\beta x}$$

Attenuation   Phase shift
with distance   with distance.

In general, α and β are very complicated expressions of R, L, C, G$\cdot$; but for the case $\frac{R}{L} = \frac{G}{C}$, we have

$$\alpha = \sqrt{RG}, \qquad \beta = \omega\sqrt{LC};$$

while for the "low loss" case: $R \ll \omega L$, $G \ll \omega C$, we have

$$\left. \begin{array}{l} \beta \approx \omega\sqrt{LC} \\[10pt] \alpha \approx \frac{1}{2}\left[R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}} \right] = \frac{1}{2}\left(\frac{R}{Z_o} + G Z_o \right). \end{array} \right\} \tag{15}$$

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Substitution of (12) and (13) in (8) and (9) shows that

$$\frac{A}{C} = -\frac{B}{D} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = Z_c \qquad (16)$$

Z_c is the characteristic impedance of the line. For the distortionless case $\frac{R}{L} = \frac{G}{C}$, we have

$$Z_c = \sqrt{\frac{L}{C}} = \sqrt{\frac{R}{G}} = Z_0$$

For the "low loss" case ($R \ll \omega L$, $G \ll \omega C$), we have

$$Z_c \approx \sqrt{\frac{L}{C}} \left[1 + j\frac{1}{2}\left(\frac{R}{\omega L} - \frac{G}{\omega C} \right) \right] \approx \sqrt{\frac{L}{C}} = Z_0 \qquad (17)$$

If the transmission line extends from $x = 0$ to $x = \infty$ and the excitation is by a generator at $x = 0$, the solution will consist of only the first term in (5) and (6) or in (12) and (13). With no loss the voltage conditions in the harmonic case are as shown below.
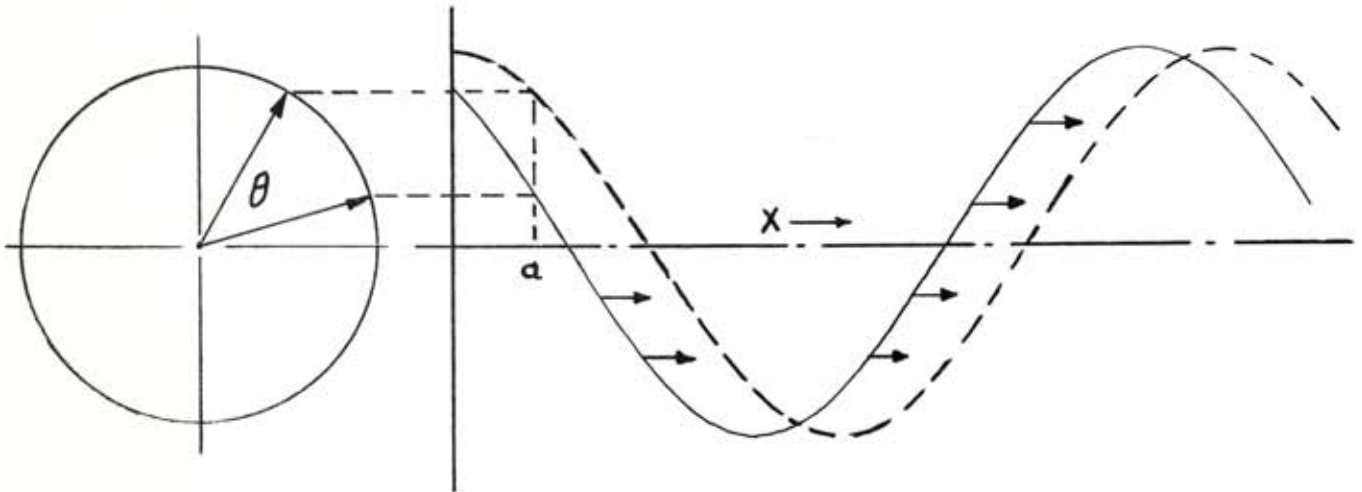
The <u>solid</u> line shows the wave now ($t = 0$).
The <u>dotted</u> line shows the wave later ($t = t_0$).
The voltage at $x = 0$ is the generator voltage.
The voltage now (solid) at $x = a$ is the generator voltage at the earlier time $t = -t_0$.
The voltage <u>later</u> (dotted) at $x = a$ is the generator voltage now. As we move from
$x = 0$ to $x = a$ the voltage we see is exactly as if we had stayed at $x = 0$ and
the generator had dropped back in phase by an angle $-\theta$. If we move a
distance λ, $-\theta = 2\pi$. Hence

$$-\theta = \frac{2\pi}{\lambda} x = \frac{2\pi f}{v} x = \frac{\omega}{v} x.$$

But $v = \dfrac{1}{\sqrt{LC}}$, so $- \theta = \omega \sqrt{LC} \ x = \beta x$.

If we look at any point on the infinite lossless line we will see the generator voltage delayed by an angle $- \theta = \beta x$. That is

$$E = E(0) \ e^{j\theta} = E(0) \ e^{-j\beta x}.$$

When the line is lossy a similar picture is obtained. The difference is that as we advance down the line the voltage is the generator voltage delayed by angle $-\theta$, and decreased in amplitude by a factor $e^{-\alpha x}$. That is

$$E(x) = E(0) \ e^{-\alpha x} \ e^{-j\beta x} = E(0) \ e^{-\alpha x}.$$

As a result we can represent the voltage (or current) for a wave traveling to the right as a vector which rotates clockwise uniformly and decreases exponentially in length with <u>increasing</u> x. The locus of the tip of this vector is a logarithmic spiral

$$E = E(0) \ e^{\left(\frac{\alpha}{\beta}\right)\theta}$$

where θ is measured counterclockwise. The ratio $\dfrac{\alpha}{\beta} \approx \dfrac{1}{\omega} \sqrt{\dfrac{RG}{LC}}$ is a measure of the lossiness of the line. For $\dfrac{\alpha}{\beta} \ll 1$. The locus is nearly circular and the loss per wavelength is small.

Similarly a wave traveling to the left can be represented as a vector which rotates clockwise and decreases exponentially with <u>decreasing</u> x.

Thus consider two voltages which happen to be in phase at $x = x_0$, as shown in the center figure below. If one voltage E_R is from a wave propagating to the right, the other E_L from a wave propagating to the left, then at $x < x_0$ the picture will be as shown in the left figure. For $x > x_0$ the picture will be as shown in the right figure.
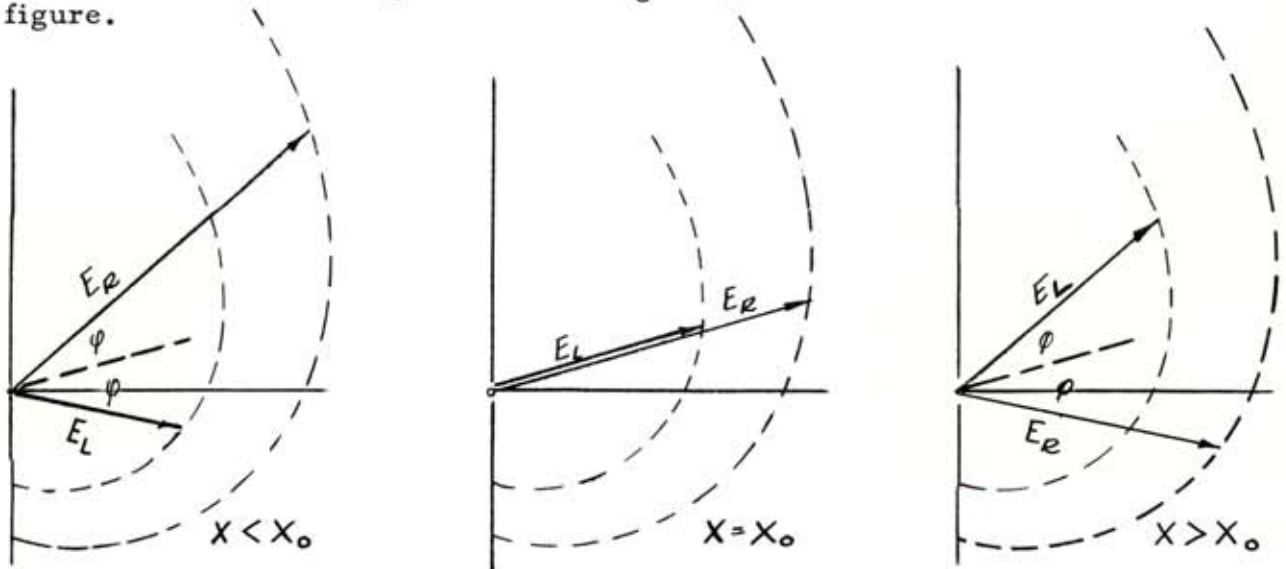
Termination, Reflection, Standing Waves

Consider any two points a-a' on the conductors of an infinite line.
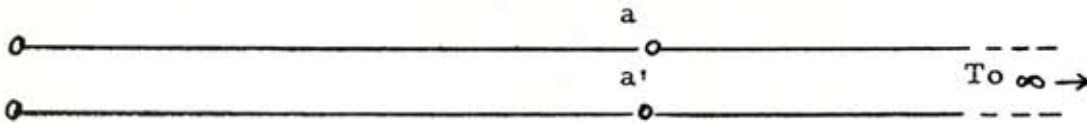
To a wave incident on these points from the left, the line to the right merely looks like an impedance Z_0 connecting a and a'. If we chop the line off at a-a' and connect an impedance Z_0 across a-a' then so far as the line to the left is concerned, Z_0 just looks like more line. Waves coming from the left will be totally absorbed by the connected impedance. We say such a line is "terminated in a matching impedance", or "matched", or simply "terminated". Regardless of length, such a line looks infinite to the generator which sees the impedance Z_0.

If we now terminate the line in an impedance Z_L, different from Z_0, a reflected wave is produced. To see how this happens, first consider the relative polarities of the incident and reflected voltages and currents. If the voltage on a conductor is positive and the current in it at the same instant is to the right, the wave is traveling to the right. If either E or I is of opposite sign, the wave is to the left. Hence the following polarities and senses are consistent for our case:
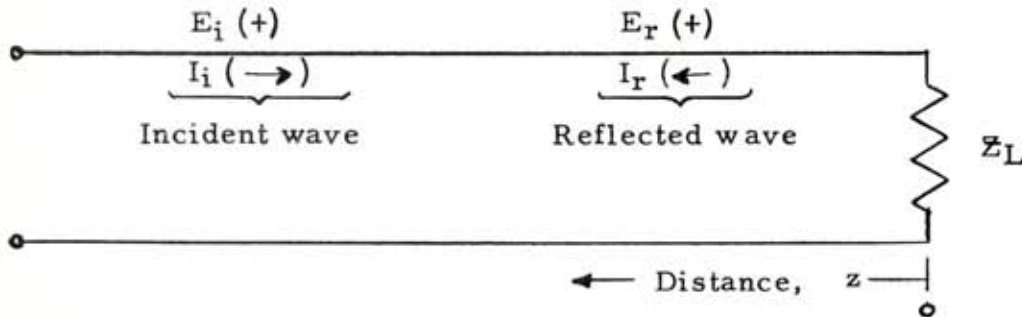
For convenience, let us measure distance from the load. Then at the load we have

$$E_L = E(0) = E_i(0) + E_r(0)$$

$$I_L = I(0) = I_i(0) - I_r(0)$$

But $Z_L = \dfrac{E_L}{I_L}$ while $\dfrac{E_i}{I_i} = \dfrac{E_r}{I_r} = Z_0$. Hence

$$Z_L = \frac{E_i(0) + E_r(0)}{I_i(0) - I_r(0)} = Z_0 \frac{1 + \dfrac{E_r(0)}{E_i(0)}}{1 - \dfrac{I_r(0)}{I_i(0)}}$$

$$\frac{Z_L}{Z_o} = \frac{1 + \dfrac{E_r(0)}{E_i(0)}}{1 - \dfrac{I_r(0)}{I_i(0)}} \qquad (18)$$

If we now let $\quad \rho = \dfrac{E_r(0)}{E_i(0)} = \dfrac{I_r(0)}{I_i(0)} \quad$ we have

$$\frac{Z_L}{Z_o} = \frac{1 + \rho}{1 - \rho} \qquad (19)$$

Or, turning things around

$$\rho = \frac{\dfrac{Z_L}{Z_o} - 1}{\dfrac{Z}{Z_o} + 1} = \frac{Z_L - Z_o}{Z_L + Z_o} \qquad (20)$$

ρ is called the <u>reflection coefficient</u>. It is the (complex) ratio of the amplitude of the reflected wave to the amplitude of the incident wave. We shall represent its magnitude by <u>r</u>. This is $\rho = r e^{j\phi}$.

The voltage at the load is

$$E_L = (1 + \rho) E_i(0) = \frac{2Z_L}{Z_L + Z_o} E_i(0)$$

The current at the load is

$$I_L = (1 - \rho) I_i(0) = \frac{2Z_o}{Z_L + Z_o} I_i(0) = \frac{2E_i(0)}{Z_L + Z_o} .$$

The ratio of reflected power to incident power is given by

$$\frac{P_r}{P_i} = |\rho|^2 = r^2$$

The <u>return loss</u> is this ratio expressed in db. Thus

$$\text{Return Loss} = -10 \log \frac{P_r}{P_i}$$

$$= -20 \log r.$$

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The remainder of the power is absorbed by the load so we have

$$\frac{P_L}{P_i} = 1 - r^2$$

The reflection loss is the ratio expressed in db. Thus

$$\underline{\text{Reflection Loss}} = 10 \log (1 - r^2) \approx 4.343 r^2 \text{ for } r \ll 1.$$

If there is reflection at the receiving end of the line, the reflected wave amplitude (current or voltage) will add vectorially to the incident wave amplitude. If there is no loss, the incident and reflected vectors merely rotate in opposite directions as we move along the line. At a point where E_i and E_r are in phase the total voltage will have the magnitude $(1 + r) E_i$. As we move toward the generator a distance Δz from this point E_i will advance in phase an amount $\beta \Delta z$ while E_r will retard the same amount. The total angle between them is thus $2\beta\Delta z$. When

$$2\beta \Delta z = \pi$$

$$\Delta z = \frac{\pi}{2\beta} = \frac{\pi V}{2\omega} = \frac{V}{4f} = \frac{\lambda}{4}$$

The two vectors will be opposed and the total voltage will have the magnitude $(1 - r) E_i$. The action repeats cyclically, maxima and minima occurring alternately every quarter wavelength. The ratio of maximum to minimum is called the standing wave ratio. Evidently:

$$SWR = \frac{1 + r}{1 - r} \qquad (21)$$

By comparing (19) and (21) we see that a resistance termination produces a standing wave ratio which is its normalized value or the reciprocal. Thus a 50^ω line terminated in a resistance of 100^ω (or 25^ω) will have a 2 to 1 SWR.

Equation (21) may be inverted to give

$$r = \frac{SWR - 1}{SWR + 1} \qquad (22)$$

Standing wave measurements thus indicate (1) the frequency and (2) the degree of match at the load.

Input Impedance of Mis-terminated Lines

The impedance looking into a line of length ℓ terminated in Z_L may be written merely by adapting equation (18) to the point $z = \ell$, rather than $z = 0$.

Thus

$$\frac{Z}{Z_o} = \frac{1 + \dfrac{E_r(\ell)}{E_i(\ell)}}{1 - \dfrac{I_r(\ell)}{I_i(\ell)}}$$

Now

$$\frac{E_r(\ell)}{E_i(\ell)} = \frac{I_r(\ell)}{I_i(\ell)} = \frac{E_r(0)}{E_i(0)} e^{-2\gamma\ell} = \rho\, e^{-2\gamma\ell}$$

So

$$\frac{Z}{Z_o} = \frac{1 + \rho e^{-2\gamma\ell}}{1 - \rho e^{-2\gamma\ell}} \qquad (23)$$

Now substitute $re^{j\phi}$ for ρ and let $\theta = \phi - 2\beta\ell$ and we find

$$\frac{Z}{Z_o} = \frac{1 + r\,e^{-2\alpha\ell}\,e^{j\theta}}{1 - r\,e^{-2\alpha\ell}\,e^{j\theta}} \qquad (24)$$

For lossless lines $\alpha = 0$ and (24) becomes

$$\frac{Z}{Z_o} = \frac{R}{Z_o} + j\frac{X}{Z_o} = \frac{1 + r\,e^{j\theta}}{1 - r\,e^{j\theta}} \qquad (25)$$

Equation (25) is the equation of the Smith Chart. The equation is of the form

$$w = \frac{1 + z}{1 - z}$$

where $w = u + jv$, and $z = x + jy$ are complex numbers. Such an equation is known as a bilinear transformation and is characterized by the fact that lines of constant u or constant v plot as circles in the z-plane. Since in our case $r = |z| \lessgtr 1$, the entire plot lies within the unit circle $r = 1$.

The Smith Chart

We see from equation (25) that the Smith Chart is essentially a plot of normalized resistance and reactance as a function of magnitude and phase of reflection coefficient. In fact:

1. Distance from the center of the chart is proportional to magnitude of reflection coefficient, r.

2. Angles about the center of the chart (read on the peripheral angle scale) are the angle of the reflection coefficient ϕ, if the plane is that of the load ($z = 0$); or the effective angle, θ, if the plane is $z = \ell$.

3. The lines of constant normalized resistance $\left(\dfrac{R}{Z_o}\right)$ and reactance $\left(\dfrac{X}{Z_o}\right)$ form a set of curvilinear coordinates from which the impedance corresponding to any magnitude and phase of reflection coefficient may be read; or vice versa.

Now the convenient thing about the Smith Chart is this: In a lossless line, as we see from eq. (25), the input impedance change with line length is equivalent to a change in the phase of the reflection coefficient only. Hence as we change line length the input impedance will describe a circular path about the center of the Smith Chart. Since

$$\Delta\theta = 2\beta\Delta\ell = \frac{2\omega}{V}\Delta\ell = 4\pi\frac{\Delta\ell}{\lambda}$$

changing ℓ by $\dfrac{\lambda}{2}$ takes us completely around the circle. For convenience, a second angle scale calibrated in terms of fractions of a wavelength is provided around the periphery. With a Smith Chart you don't need tables of hyperbolic functions to compute transmission line problems; all you need is a ruler and compass. (In fact, you usually don't need the compass.) This sounds easy enough, let's try an example.

Example 1: A 50^ω line is terminated in a resistance of 30^ω in series with a capacitive reactance of 40^ω.

a) What is the reflection coefficient?
b) What is the SWR?
c) What is the input impedance if $\ell = 0.1\lambda$?
d) What lengths of line make the input impedance resistive and what is the resistance?

Solution: $\dfrac{30 - j40}{50}$ $0.6 - j0.8$ so we enter the chart at $\dfrac{R}{Z_o} = 0.6$, $\dfrac{X}{Z_o} = 0.8$, marking point A.

a) The distance of point to the center is 0.5. Radial line through A to angle scale shows $\phi = -90^\circ$. Thus $\rho = 0.5 \underline{|-90^\circ}$

b) Draw the circle of constant $r = 0.5$ through point A. This circle intersects the $\dfrac{X}{Z_o} = 0$ line on the $\dfrac{R}{Z_o} > 1$ side at $\dfrac{R}{Z_o} = 3$ (Point B). This is the SWR. (This is the $\dfrac{R}{Z_o}$ which would produce the same r as the actual load, and for $\dfrac{R}{Z_o} > 1$, $\dfrac{R}{Z_o} = $ SWR).

c) Radius through A intersects wavelength scale ("toward generator" scale) at 0.375λ. (This is just an arbitrary starting point. On fancy Smith charts, wavelength scale can be rotated to put index here.) Proceed 0.1λ toward generator - you're going away from load, aren't you? -- to 0.475. Draw radius. Intersects circle (Point C) at 0.336 - j 0.144 or 16.8 - j 7.2 ohms.

d) Impedance will be real at Points D and B. Takes $0.5\lambda - 0.375\lambda = \dfrac{\lambda}{8}$ to get to D from A. Another $\dfrac{\lambda}{4}$ to get to B. Resistance at D is $50 \times 0.333 \ldots = 16.67$ ohms. Resistance at B is $50 \times 3 = 150^{\omega}$.

Referring to equation (25) we see that changing θ by π inverts the expression. Thus points diametrically opposite each other on the Smith Chart are inverses of each other with respect to Z_0, i.e. $\dfrac{Z_1}{Z_0} = \dfrac{Z_0}{Z_2}$ or $Z_1 Z_2 = Z_0^2$. This illustrates the impedance inverting property of a quarter wave $\left(\text{or } \dfrac{(2n-1)\lambda}{4}\right)$ line, since it takes a quarter wavelength to go halfway around the chart. Further, since

$$\frac{Z_0}{Z} = \frac{Y}{Y_0}$$

the point diametrically opposite a given normalized impedance point is the normalized admittance representation. Thus the Smith Chart can be used equally well on an impedance or admittance basis. In the latter case one merely uses the $\dfrac{R}{Z_0}$ scale as $\dfrac{G}{Y_0}$ and the $\dfrac{X}{Z_0}$ scale as $\dfrac{B}{Y_0}$.

The voltage maxima along a line occur at the point where $\dfrac{Z}{Z_0}$ is greatest, i.e., where ρ is real and positive; the minima where $\dfrac{Z}{Z_0}$ is least, i.e., where ρ is real and negative. Thus the maxima are where the r-circle intersects the $\dfrac{X}{Z_0} = 0$ line where $\dfrac{R}{Z_0} > 1$ and the minima at the intersection where $\dfrac{R}{Z_0} < 1$.

A standing wave measurement suffices to determine the magnitude of the reflection coefficient, r, of an unknown load. To complete the picture the phase (referred to some reference plane) must also be found. This can be done by measuring the position of the minima of the SW pattern with the unknown, relative to their positions with a load having known reflection phase, and is usually carried out as follows:

1. Locate a voltage minimum with the actual load. (Any minimum near the middle of the slotted section will do.)

2. Replace the load with short circuit.

3. Locate a nearby new null and express in wavelengths the distance you moved (toward generator or toward load) from the old minimum. Draw this radial line on the Smith Chart, using the wavelength scales (with their zeros opposite $\frac{Z}{Z_o} = 0$).

4. Draw the circle representing the measured SWR and read the impedance where it intersects the radial line.

The impedance so obtained is referred to the plane of the short. The reason the method works may be seen from the drawing below.

Pattern with Short   Pattern with Load   Plane of Load

L   L   L   L

d_1 d_2 d_2 d_1
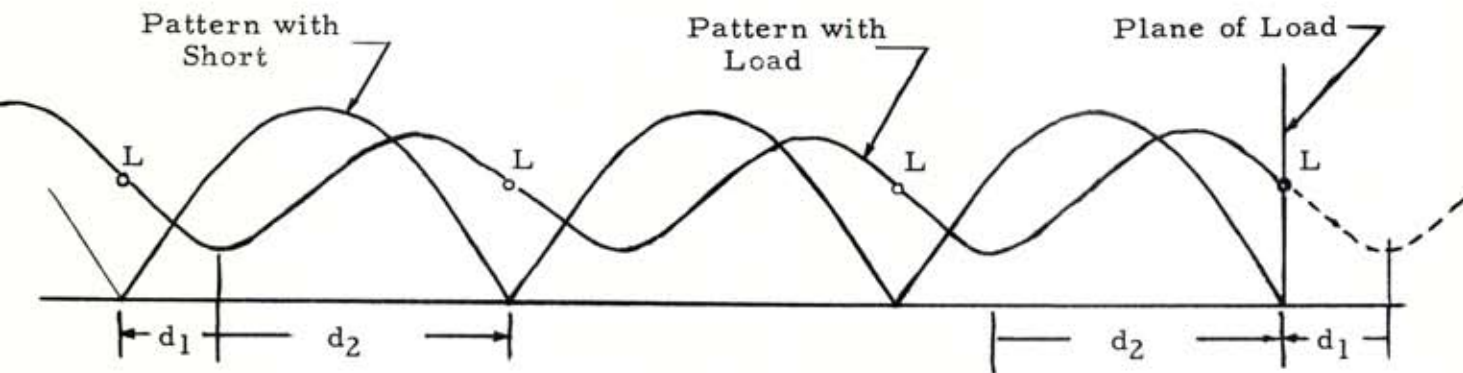
Input Impedance = Load Impedance at Points Marked "L".

Since the input impedance of the line is equal to the load impedance for a line any number of half wavelengths long, and since these half wavelength multiples occur at the nulls with a short, the input impedance will equal the load impedance either a distance d_1 toward the generator or a distance d_2 toward the load from the observed minimum. Now the input impedance at the observed minimum occurs on the $\frac{X}{Z_o} = 0$ line for $\frac{R}{Z_o} < 1$ on the chart. This happens to be the zero axis for the wavelength scale (as printed) so all we do is go toward the generator a distance d_1 on our r-circle or toward the load a distance d_2. Since $d_1 + d_2 = \frac{\lambda}{2}$ we will end up at the same point on the chart.

Additional radial scales are often provided on the Smith Chart. Some of these and their uses are listed below:

1. Voltage or Current Limits

(Max. and Min.) These are scales of $\sqrt{SWR}$ and $\dfrac{1}{\sqrt{SWR}}$ respectively. They are used to find the maximum and minimum voltages or currents on a line when the line impedance and load power are known, using the relations:

$$E_{max} = \sqrt{PZ_o \,(SWR)} \qquad I_{max} = \sqrt{P(SWR)/\,Z_o}$$

$$E_{min} = \sqrt{PZ_o/\,(SWR)} \qquad I_{min} = \sqrt{P/Z_o\,(SWR)}$$

2. Voltage or Current Ratios -

$\left(\dfrac{Max.}{Min.}\text{ and db}\right)$ These are scales of SWR and 20 log SWR.

3. Reflection Loss (db) -

This is a scale of $-10 \log (1 - r^2)$, i.e. the load mismatch loss in db.

4. S. W. Loss Coefficient -

This is a scale of $\dfrac{1 + r^2}{1 - r^2}$ which is the factor by which the actual power dissipation in the line is increased by the SWR.

5. "1 db Steps" (or 1/2 Return Loss) -

This is a scale which can be used to compute or to take account of line loss as explained below. If each step is taken as 2 db, starting with the outside mark as 0 db, the scale becomes a scale of return loss $(-10 \log r^2)$.

Use of Smith Chart with Lossy Lines

So far our discussion of the Smith Chart has assumed a lossless line. The locus of input impedance as line length varies is then a circle, as we have seen.

Referring to equation (24) we see that if $\alpha \neq 0$ then the effective reflection coefficient not only changes in angle (θ) but also decreases exponentially in magnitude as we progress toward the generator. The Smith Chart can still be used to convert between input and load impedances, but the loci along which impedances move with l are now logarithmic spirals, not circles. However, if the attenuation per wavelength is known the process is not difficult.

The losses are most simply taken into account with the aid of an additional radial scale marked off in 1 db steps, the use of which is best illustrated by example. Suppose the line in Example 1 had a loss of 5 db per wavelength. Then the answer to part (c) would be obtained as follows.

1. We would first pretend the line had no loss and obtain point C, as before.

2. Since we have moved away from the load, the effect of line loss will be to make the input impedance nearer the characteristic impedance than would be the case without loss. The total intervening attenuation has been 0.1 wavelength x 5 db per wavelength or 0.5 db. Accordingly, we move inward radially from point C by 0.5 db on the 1 db steps scale to obtain point E. This point is the normalized input impedance with the lossy line.

Since the 1 db steps are farthest apart for $r \approx 1$, we see that corrections for loss will have the greatest effect on impedances when the SWR is high.

The attenuation of a lossy line can be determined by a measurement of SWR of shorted length of line. Since the reflection coefficient of the load is unity, the reflection coefficient at $z = \ell$ will be $r = e^{-2\alpha\ell}$. Hence we have

$$\alpha\ell = -\frac{1}{2} \ln r = \frac{1}{2} \ln \left(\frac{SWR + 1}{SWR - 1}\right) \quad \text{(nepers)}$$

$$= -4.343 \log r = 4.343 \log \left(\frac{SWR + 1}{SWR - 1}\right) \quad \text{(db)}$$

This quantity (in db) can be read directly from the "1 db steps" radial scale as the number of db from $r = 1$ (SWR $= \infty$) to the point opposite the measured r (or SWR).

The load need not be a short, though this is best. With less than unity reflection at the load one takes the difference on the db steps scale between the point for r (or SWR) as measured at the load and the point for r (or SWR) as measured through the line.

The measurement of attenuation using the "db steps" scale is not very accurate for very low or very high attenuation. However, with a slotted line, the expression

$$\alpha\ell = 4.343 \log \left(\frac{SWR + 1}{SWR - 1}\right) \quad \text{(db)}$$

is quite accurate for low attenuations. For high attenuations a transmission measurement is preferable.

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